

Homework assignment

L5: Regression models for item description

Assignment date: 05.11.2019
Deadline: 11.11.2019 23:59
Slides: <http://www.cs.cas.cz/martinkova/NMST570>
Note: Send answers to hladka@cs.cas.cz
Name:

1 Interpretation of logistic regression models

Ex. 1.1 Logistic regression model for probability of correct answer on total scores X to given item is given by:

$$P(Y = 1|X, b_0, b_1) = \frac{\exp(b_0 + b_1 X)}{1 + \exp(b_0 + b_1 X)}. \quad (1)$$

Consider parameters $b_0 = -5$ and $b_1 = 0.5$.

1. How do you interpret these parameters? [0.5]
2. Fill in the table below with probabilities of correct answer given by (1). [1]

Total score X	0	5	10	15	20
$P(Y = 1 X, b_0, b_1)$					

3. Using table above, try to sketch curve for probability of correct answer (1) [0.75].

Ex. 1.2 Consider logistic regression model on standardized total scores Z , that is

$$P(Y = 1|Z, \tilde{b}_0, \tilde{b}_1) = \frac{\exp(\tilde{b}_0 + \tilde{b}_1 Z)}{1 + \exp(\tilde{b}_0 + \tilde{b}_1 Z)}. \quad (2)$$

1. What are the values of parameters \tilde{b}_0 and \tilde{b}_1 in model (2) when values of parameters b_0 and b_1 are given by Exercise 1.1 and mean of total scores X is $M = 10$ and their standard deviation is $SD = 3$? Provide whole calculation. [1.25]
 HINT: Use $Z = (X - M)/SD$ for arbitrary X and solve $P(Y = 1|X, b_0, b_1) = P(Y = 1|Z, \tilde{b}_0, \tilde{b}_1)$

2. Use calculated values of \tilde{b}_0 and \tilde{b}_1 and interpret them. [0.5]

Ex. 1.3 Consider logistic regression model on standardized total scores Z with IRT parameterization, that is

$$P(Y = 1|Z, a, b) = \frac{\exp(a(Z - b))}{1 + \exp(a(Z - b))}. \quad (3)$$

1. What are the values of parameters a and b in model (3) when values of parameters \tilde{b}_0 and \tilde{b}_1 are given by Exercise 1.2.1? Provide whole calculation. [0.75]
2. Use calculated values of a and b and interpret them. [0.5]

2 Extensions of logistic regression models

Ex. 2.1 Consider non-linear extension of logistic regression model on Z -scores with IRT parameterization, that is

$$P(Y = 1|Z, a, b, c, d) = c + (d - c) \frac{\exp(a(Z - b))}{1 + \exp(a(Z - b))}, \quad (4)$$

with parameters $a = 1.5$, $b = 0$, $c = 0.3$, and $d = 0.9$.

1. How do you interpret parameters c and d ? [0.5]
2. Sketch curve for probability of correct answer and show how it is related to parameters c and d . Describe how is now defined parameter b compare to the case when $c = 0$ and $d = 1$. [1]

Ex. 2.2 Consider multinomial model, that is

$$P(Y = k|Z, a_k, b_k) = \frac{\exp(a_k(Z - b_k))}{\sum_{j=1}^K \exp(a_j(Z - b_j))}, \quad k = 0, 1, 2, \quad (5)$$

where $a_0 = 0$, $b_0 = 0$, $a_1 = 1.5$, $b_1 = -0.6$, $a_2 = 2$ and $b_2 = 0$.

1. Fill table below with probabilities of $k = 0$, $k = 1$ and $k = 2$ for different levels of Z . [1.25]

Z	-1.8	-0.6	0	0.6	1.8
$P(Y = 0)$					
$P(Y = 1)$					
$P(Y = 2)$					

2. Using table above, sketch curves for probabilities of $k = 0$, $k = 1$, and $k = 2$ in one plot. For what levels of Z do the curves cross? [1]

3 ShinyItemAnalysis

Run `ShinyItemAnalysis` (online or locally) and change data to HCL.

1. Fit various logistic regression models (Logistic IRT Z, Logistic 3P IRT Z and Logistic 4P IRT Z) for item 4. Which model does fit the best using likelihood ratio test? Interpret parameter estimates for the best model. (Regression/Model Comparison) [0.5]
2. Fit various logistic regression models (Logistic IRT Z, Logistic 3P IRT Z and Logistic 4P IRT Z) for item 19. Which model does fit the best using likelihood ratio test? Interpret parameter estimates for the best model. [0.5]

4 Provide feedback

Here you can provide feedback on lecture, lab session and/or materials (slides, HW assignment, ShinyItemAnalysis manual) [1pt bonus] :